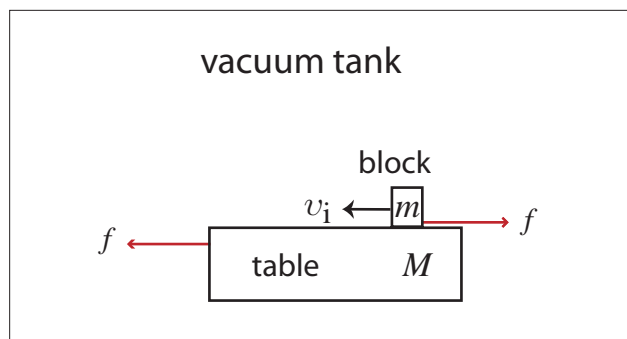


# Thermodynamics of a Block Sliding Across a Frictional Surface

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The following idealized problem is intended to illustrate some basic thermodynamic concepts involved in kinetic friction. A block of mass  $m$  is sliding on top of a frictional,<sup>1</sup> flat-topped table of mass  $M$ . The table is magnetically levitated, so that it can move without thermal contact and friction across a horizontal floor. The table is initially stationary, while the block has initial speed  $v_i$  and slides to rest relative to the table. The block and table are inside a large vacuum tank, so there is no air resistance, buoyancy, nor thermal losses to the atmosphere. Furthermore the inner surface of the vacuum



**Fig. 1. The initial configuration as seen in the lab frame.** There are equal and opposite kinetic frictional forces  $f$  on the block and table, in accord with Newton's third law. Heat transfers from the block and table to any other objects in the environment are obviated as follows. The tank is evacuated to eliminate convection. The table is magnetically levitated to prevent thermal conduction to the tank. Finally, the interior surface of the vacuum vessel is perfectly reflecting (so that it has zero absorptivity and emissivity by Kirchhoff's law) to avoid radiative exchanges of energy.

tank is a perfect mirror so that the tank does not radiatively exchange heat with the block and table. The block and table are homogeneous, both initially have temperature  $T_i$ , and they each have large thermal conductivities so that they rapidly attain a common final temperature  $T_f$  after the block has come to rest. The specific heat capacity of the block is  $c_b$  and that of the table is  $c_t$ , and these heat capacities are assumed to be temperature independent over the range of temperatures that arises in this problem. (a) Find the common final speed  $v_f$  and temperature  $T_f$  of the block and table. (b) Find the changes in the bulk kinetic energies  $K$  (in the center-of-mass frame of the isolated block-table system), the internal energies  $U$ , and the entropies  $S$  of the block and table. (c) Discuss the first and second laws of thermodynamics in connection with these results.

## Solution

The setup is sketched in Fig. 1. The common final speed in the lab frame can be found using conservation of momentum,

$$mv_i + 0 = (m + M)v_f \Rightarrow v_f = \frac{m}{m + M}v_i. \quad (1)$$

(The process amounts to a perfectly inelastic collision.) In the center-of-mass frame, the initial speeds of the block and table are  $v_i - v_f$  and  $v_f$  respectively, and their final speeds are zero. The change in the bulk kinetic energy of the block is therefore

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$$\Delta K_b = 0 - \frac{1}{2}m \left( v_i - \frac{m}{m+M} v_i \right)^2 \quad (2)$$

$$= -\frac{1}{2}m \left( \frac{M}{m+M} v_i \right)^2,$$

while the corresponding change for the table is

$$\Delta K_t = -\frac{1}{2}M \left( \frac{m}{m+M} v_i \right)^2, \quad (3)$$

both of which are negative because the block and table each lose all of their kinetic energy in this frame. The sum of these losses in mechanical energy is transformed into the sum of the internal energies they gain since the system is mechanically and thermally isolated,

$$\Delta U_b + \Delta U_t = -(\Delta K_b + \Delta K_t) = \frac{1}{2} \mu v_i^2, \quad (4)$$

where  $\mu = mM / (m + M)$  is the reduced mass of the block-table combination. During and immediately after the sliding process, the block and table have nonuniform temperatures (their facing surfaces being hotter than their more removed sections). But by assumption, they will quickly equilibrate to a uniform and common final temperature. The resulting temperature rise of the block and table is determined by the changes in their internal energies,

$$\Delta U_b = mc_b (T_f - T_i) \text{ and } \Delta U_t = Mc_t (T_f - T_i). \quad (5)$$

Adding these and substituting into Eq. (4) gives the common final temperature of

$$T_f = T_i + \frac{\mu v_i^2}{2(mc_b + Mc_t)}. \quad (6)$$

Inserting this result back into Eq. (5) implies that the changes in internal energies of the block and table are

$$\Delta U_b = \frac{\mu v_i^2}{2} \left( \frac{mc_b}{mc_b + Mc_t} \right)$$

and (7)

$$\Delta U_t = \frac{\mu v_i^2}{2} \left( \frac{Mc_t}{mc_b + Mc_t} \right).$$

Since the heat capacities are assumed to be temperature independent, the changes in entropies can be calculated as

$$\Delta S_b = \int_{T_i}^{T_f} \frac{mc_b dT}{T} = mc_b \ln \frac{T_f}{T_i}$$

$$= mc_b \ln \left[ 1 + \frac{\mu v_i^2}{2T_i(mc_b + Mc_t)} \right] \quad (8)$$

and

$$\Delta S_t = Mc_t \ln \left[ 1 + \frac{\mu v_i^2}{2T_i(mc_b + Mc_t)} \right]. \quad (9)$$

If the final temperature is only slightly larger than the initial temperature, as one would expect for a real block and table and a reasonable initial speed, then the total change in entropy of the block and table can be obtained using the expansion  $\ln(1 + x) \approx x$  for  $|x| \ll 1$ . In that case, Eqs. (8) and (9) imply that the total entropy change of the system is

$$\Delta S \equiv \Delta S_b + \Delta S_t \approx \frac{\mu v_i^2}{2T_i}, \quad (10)$$

which is simply the total gain in internal energy from Eq. (4) divided by the approximately constant temperature. Note that  $\Delta S > 0$ , which is the second law of thermodynamics for this irreversible process.

The net change in total energy ( $E = K + U$ ) of the block is

$$\Delta E_b = \Delta K_b + \Delta U_b$$

$$= \frac{\mu v_i^2}{2} \left( -\frac{M}{m+M} + \frac{mc_b}{mc_b + Mc_t} \right), \quad (11)$$

while that of the table is

$$\Delta E_t = \Delta K_t + \Delta U_t = \frac{\mu v_i^2}{2} \left( -\frac{m}{m+M} + \frac{Mc_t}{mc_b + Mc_t} \right). \quad (12)$$

By rearranging terms, one finds that  $\Delta E_b = -\Delta E_t$ , consistent with Eq. (4). That is, the energy lost by the block is equal to the energy gained by the table, which is the first law of thermodynamics for this isolated system.

## Heat and Work

The concepts of work ( $W$ ) and heat ( $Q$ ) for the interaction between the block and table do not appear in the solution above. All thermodynamic quantities of interest can be calculated straightforwardly without them. This illustrates the idea that it is not necessary to

attempt to distinguish  $W$  from  $Q$  in irreversible thermodynamics.<sup>2,3</sup>

Traditionally, the first law of thermodynamics is written in the form  $W + Q = \Delta E$ . Here  $Q$  represents energy transferred from hot objects in the surroundings to the cooler system of interest by thermal conduction (including convection) or by blackbody radiation. On the other hand,  $W$  is the energy transferred from external agents of bulk forces (such as springs, muscles, gravitational or electric fields, or high-pressure gases) that displace the system in part or overall. Regardless of whether the energy transfer from object A in the surroundings to object B in the system is via heat or work, energy conservation *for each individual channel of interaction* requires that the quantity of energy gained by object B via that particular channel must be equal to the corresponding energy lost by object A. For example, if a hot spring pushes on a cold hockey puck (with both assumed to be isolated from the ice and atmosphere for simplicity), the kinetic energy gained by the puck from the mechanical push is equal to the loss in potential energy of the spring, and *separately* the internal energy gained by the puck from thermal conduction is equal to the thermal energy the spring loses, giving rise to temperature changes of the puck and spring.

From this point of view, there is nothing in the preceding solution that we can cleanly identify with  $W$  and  $Q$  separately. While it is tempting<sup>4–6</sup> to identify  $W$  with  $\Delta K$  and  $Q$  with  $\Delta U$ , that is inconsistent with the facts that  $\Delta K_b \neq -\Delta K_t$  and  $\Delta U_b \neq -\Delta U_t$ . Intuitively, it is clear that as the block moves along, its sliding surface will progressively get hotter. But it is constantly traveling onto fresh, cooler portions of the tabletop. Therefore, the block will conduct heat to the table. During any small interval of time  $dt$ , the block will lose some small quantity of energy  $dE_c$  by thermal conduction (and radiation), and the table will gain that same amount of energy  $dE_c$ . Integrating over time from  $t = 0$  until long after the block has stopped moving relative to the table and thermal equilibrium has been restored, we can in principle compute  $\Delta E_c$ , which we identify as  $Q$ , representing the total amount of energy lost by the block and gained by the table via thermal channels due to the temporary temperature differences between their surfaces in contact (taking the system to be the table and the surroundings to be

the block). Then  $\Delta E_t - \Delta E_c$  must represent the energy transferred athermally<sup>7</sup> to the table as a result of the mechanical deformations of small asperities on the surface of the table by the block and which we therefore identify as  $W$ , the net work that the block does on the table.

In practice, individual calculation of  $Q$  and  $W$  therefore requires that one determine  $dE_c$  instant by instant from a microscopic model of the (nonuniform) temperature distributions of the block and table during and immediately after sliding. In general this is a difficult problem, requiring finite-element computer simulations, although Sherwood and Bernard<sup>8</sup> have treated some simple, highly symmetric cases and explicitly explained why  $W$  is not equal to  $\Delta K$ . On the other hand, we can easily determine the sum  $Q + W$ , which represents the total energy transferred from the block to the table. It is given by the right-hand side of Eq. (12), which as already noted is properly equal and opposite to Eq. (11).

## Concluding Remarks

Some authors have suggested abandoning the concepts of heat and work in thermodynamics entirely.<sup>9</sup> Others suggest retaining them but emphasizing that the first law of thermodynamics should be written more broadly as  $\sum T = \Delta E$ , where  $T$  refers to any of a variety of modes of energy transfer including mass transfer, mechanical waves, and nonthermal radiation, which cannot be neatly categorized as either heat or work.<sup>10</sup> In my opinion, while  $Q$  and  $W$  have important roles in the introductory teaching of the reversible thermodynamics of simple systems such as ideal gases, students should eventually be brought to realize that it is not always convenient nor necessary to categorize all channels of energy transfer as either “heat” or “work.” As the present problem illustrates, we can compute quantities of physical interest, such as final speeds, temperatures, and energy and entropy changes, without involving the concepts of work and heat.

## References

1. Such a surface is traditionally referred to as “rough.” However, smooth surfaces can also exhibit large friction (such as two unoxidized metal surfaces in contact), as discussed in E. Corpuz and N.S. Rebbello, “Research and instructional strategies for student modeling of microscopic friction,” (AAPT Summer Meeting, 2006). Also

see F.P. Bowden and D. Tabor, *The Friction and Lubrication of Solids* (Oxford Univ. Press, London, 1954 & 1964), Parts I & II.

2. C.E. Mungan, "Why distinguish work from heat?" *Forum on Education of the American Physical Society Newsletter*, Fall 2005, p. 3.
3. J.S. Denker, "The laws of thermodynamics," online at <http://www.av8n.com/physics/thermo-laws.htm>.
4. The change in bulk translational energy  $\Delta K$  is equal to the pseudowork, as in the work-kinetic energy theorem in mechanics and should not be confused with first-law work, which represents an energy transfer. For example, if a girl on roller skates pushes off from a rigid wall, she gains kinetic energy. But the wall has not done work on the girl in the sense of transferring energy to her! Her gain in kinetic energy is at the expense of internal chemical energy from her muscles. Choosing the girl to be the system and the wall to be the surroundings,  $\Delta K$  is positive,  $\Delta U$  is negative (specifically equal and opposite to  $\Delta K$ ), and  $W$ ,  $Q$ , and  $\Delta E = \Delta K + \Delta U$  are all zero. For a recent review of this issue, see C.E. Mungan, "A primer on work-energy relationships for introductory physics," *Phys. Teach.* **43**, 10–16 (Jan. 2005).
5. A.B. Arons, "Developing the energy concepts in introductory physics," *Phys. Teach.* **27**, 506–517 (Oct. 1989).
6. R.P. Bauman, "Physics that textbook writers usually get wrong," *Phys. Teach.* **30**, 264–269 (May 1992) and 353–356 (Sept. 1992).
7. This is alternatively called the "adiabatic work" in M.W. Zemansky and R.H. Dittman, *Heat and Thermodynamics*, 7th ed. (McGraw-Hill, New York, 1997), Chap. 4.
8. B.A. Sherwood and W.H. Bernard, "Work and heat transfer in the presence of sliding friction," *Am. J. Phys.* **52**, 1001–1007 (Nov. 1984).
9. G.M. Barrow, "Thermodynamics should be built on energy—not on heat and work," *J. Chem. Educ.* **65**, 122–125 (Feb. 1988) and H.R. Kemp, "The concept of energy without heat or work," *Phys. Educ.* **19**, 234–240 (Sept. 1984).
10. R.A. Serway and J.W. Jewett Jr., *Principles of Physics*, 4th ed. (Thomson Brooks/Cole, Belmont CA, 2006), Chap. 6.

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**Carl Mungan** is a civilian tenured professor. This article evolved out of discussions with Al Saperstein. The goal was to devise a problem involving mechanics and thermodynamics for which the explicit introduction of either heat or work would detract from the solution.

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